

11.4.1.  $\sin A$ ,  $\sec A$  ಮತ್ತು  $\tan A$  ಈ ತ್ರಿಕೋನಮಿತಿ ಅನುಪಾತಗಳನ್ನು  $\cot A$  ರೂಪದಲ್ಲಿ ವ್ಯಕ್ತಪಡಿಸಿ.

$\sin A$	$\sec A$	$\cot A$
$1 = \sin^2 A + \cos^2 A$ ಇದರ ಎರಡೂ ಬದಿಯನ್ನು $\sin^2 A$ ಯಿಂದ ಭಾಗಿಸಿ $\frac{1}{\sin^2 A} = 1 + \frac{\cos^2 A}{\sin^2 A} = 1 + \cot^2 A$ $\sin^2 A = \frac{1}{1 + \cot^2 A} \therefore \sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$	$1 = \sin^2 A + \cos^2 A$ ಇದರ ಎರಡೂ ಬದಿಯನ್ನು $\cos^2 A$ ಯಿಂದ ಭಾಗಿಸಿ $\frac{1}{\cos^2 A} = \frac{\sin^2 A}{\cos^2 A} + 1$ $\frac{1}{\cos^2 A} = \tan^2 A + 1 = \frac{1}{\cot^2 A} + 1$ $\sec^2 A = \frac{\cot^2 A + 1}{\cot^2 A} \therefore \sec A = \pm \frac{\sqrt{\cot^2 A + 1}}{\cot A}$	$\tan A = \frac{1}{\cot A}$

11.4.2.  $\angle A$  ದ ಎಲ್ಲಾ ತ್ರಿಕೋನಮಿತಿ ಅನುಪಾತಗಳನ್ನು  $\sec A$  ರೂಪದಲ್ಲಿ ಬರೆಯಿರಿ

$\sin A$ & $\operatorname{cosec} A =$	$\cos A =$	$\tan A$ & $\cot A =$
$\sin^2 A = 1 - \cos^2 A$ $= 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$ $\therefore \sin A = \pm \frac{\sqrt{\sec^2 A - 1}}{\sec A}$ $\Rightarrow \operatorname{cosec} A = \frac{1}{\sin A} = \pm \frac{\sec A}{\sqrt{\sec^2 A - 1}}$	$\frac{1}{\sec A}$	$1 = \sin^2 A + \cos^2 A$ $\frac{1}{\cos^2 A} = \frac{\sin^2 A}{\cos^2 A} + 1$ $\sec^2 A = \tan^2 A + 1$ $\Rightarrow \tan^2 A = \sec^2 A - 1$ $\tan A = \sqrt{\sec^2 A - 1} \Rightarrow \cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$

ಪ್ರಶ್ನೆ	ಬೆಲೆಯನ್ನು ಕಂಡುಹಿಡಿಯಿರಿ/ಉತ್ತರಿಸಿ
11.4.5.(i)	$\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ} = ??$ $\sin^2 63^\circ + \sin^2 27^\circ = \sin^2 (90^\circ - 27^\circ) + \sin^2 27^\circ = \cos^2 27^\circ + \sin^2 27^\circ = 1 \text{ -----(1)}$ $\cos^2 17^\circ + \cos^2 73^\circ = \cos^2 (90^\circ - 73^\circ) + \cos^2 73^\circ = \sin^2 73^\circ + \cos^2 73^\circ = 1 \text{ -----(2)}$ $(1) \div (2) \Rightarrow \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ} = 1$
11.4.5.(ii)	$\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ = ??$ $= \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$ $= \sin 25^\circ \cos (90^\circ - 25^\circ) + \cos 25^\circ \sin (90^\circ - 25^\circ)$ $= \sin 25^\circ \sin 25^\circ + \cos 25^\circ \cos 25^\circ = \sin^2 25^\circ + \cos^2 25^\circ = 1$
11.4.4.(i)	$9 \sec^2 A - 9 \tan^2 A = ??$ $= 9 \left( \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A} \right) = 9 \left( \frac{1 - \sin^2 A}{\cos^2 A} \right) = 9 \left( \frac{\cos^2 A}{\cos^2 A} \right) = 9$
11.4.4.(ii)	$(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = ??$ $= (1 + \tan \theta + \sec \theta) + (\cot \theta + \cot \theta * \tan \theta + \cot \theta . \sec \theta) - (\operatorname{cosec} \theta + \operatorname{cosec} \theta . \tan \theta + \operatorname{cosec} \theta . \sec \theta)$ $= (1 + \tan \theta + \sec \theta) + (\cot \theta + 1 + \operatorname{cosec} \theta) - \operatorname{cosec} \theta - \sec \theta - \operatorname{cosec} \theta . \sec \theta$ $[\because \cot \theta . \sec \theta = \frac{\cos \theta}{\sin \theta} * \frac{1}{\cos \theta} = \operatorname{cosec} \theta \text{ \& } \operatorname{cosec} \theta . \tan \theta = \frac{1}{\sin \theta} * \frac{\sin \theta}{\cos \theta} = \sec \theta]$ $= 2 + \tan \theta + \cot \theta - \operatorname{cosec} \theta . \sec \theta$ $= 2 + \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} - \operatorname{cosec} \theta . \sec \theta = 2 + \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta * \sin \theta} - \operatorname{cosec} \theta . \sec \theta$ $= 2 + \frac{1}{\cos \theta * \sin \theta} - \operatorname{cosec} \theta . \sec \theta = 2 + \sec \theta . \operatorname{cosec} \theta - \operatorname{cosec} \theta . \sec \theta = 2$
11.4.4.(iii)	$\frac{1 + \tan^2 A}{1 + \cot^2 A} = ??$ $1 + \cot^2 A = 1 + \frac{1}{\tan^2 A} = \frac{\tan^2 A + 1}{\tan^2 A}$ $\frac{1 + \tan^2 A}{1 + \cot^2 A} = (1 + \tan^2 A) * \left\{ \frac{\tan^2 A}{1 + \tan^2 A} \right\} = \tan^2 A$

$$11.4.5.(i) (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{(1 - \cos \theta)}{(1 + \cos \theta)} \text{ ಎಂದು ಸಾಧಿಸಿ}$$

$$(\operatorname{cosec} \theta - \cot \theta)^2 = \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 = \left( \frac{1 - \cos \theta}{\sin \theta} \right)^2 = \frac{(1 - \cos \theta)^2}{\sin^2 \theta} = \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} = \frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{(1 - \cos \theta)}{(1 + \cos \theta)}$$

$$11.4.5.(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A \text{ ಎಂದು ಸಾಧಿಸಿ}$$

$$\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) * \cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A) * \cos A} = \frac{1 + 1 + 2 \sin A}{(1 + \sin A) * \cos A} = \frac{2(1 + \sin A)}{(1 + \sin A) * \cos A} = \frac{2}{\cos A} = 2 \sec A$$

$$11.4.5.(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \operatorname{cosec} \theta \cdot \sec \theta \text{ ಎಂದು ಸಾಧಿಸಿ}$$

$$\text{LHS} = \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = \frac{\tan \theta * \tan \theta}{\tan \theta - 1} + \frac{\cot \theta}{1 - \tan \theta} \left( \because 1 - \cot \theta = 1 - \frac{1}{\tan \theta} = \frac{\tan \theta - 1}{\tan \theta} \Rightarrow \frac{1}{1 - \cot \theta} = \frac{\tan \theta}{\tan \theta - 1} \right)$$

$$= \frac{\tan^2 \theta - \cot \theta}{\tan \theta - 1} = \frac{\tan^2 \theta - \frac{1}{\tan \theta}}{\tan \theta - 1} = \frac{\tan^3 \theta - 1}{\tan \theta (\tan \theta - 1)} = \frac{(\tan \theta - 1)(\tan^2 \theta + 1 + \tan \theta)}{\tan \theta (\tan \theta - 1)} = \frac{(\tan^2 \theta + 1 + \tan \theta)}{\tan \theta} = \tan \theta + \frac{1}{\tan \theta} + 1$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} + 1 = 1 + \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta * \sin \theta} = 1 + \frac{1}{\cos \theta * \sin \theta} = 1 + \operatorname{cosec} \theta \cdot \sec \theta$$

$$11.4.5.(iv) \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{(1 - \cos A)} \text{ ಎಂದು ಸಾಧಿಸಿ}$$

$$\text{LHS} = 1 + \frac{1}{\sec A} = 1 + \cos A = \frac{(1 + \cos A)(1 - \cos A)}{(1 - \cos A)} = \frac{(1 - \cos^2 A)}{(1 - \cos A)} = \frac{\sin^2 A}{(1 - \cos A)}$$

11.4.5.(v)  $1 = \operatorname{cosec}^2 A - \cot^2 A$  ಎನ್ನುವ ನಿತ್ಯ ಸಮೀಕರಣ ಉಪಯೋಗಿಸಿ  $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$  ಎಂದು ಸಾಧಿಸಿ

$$\begin{aligned} \text{LHS} &= \frac{\frac{\cos A - \sin A + 1}{\sin A}}{\frac{\cos A + \sin A - 1}{\sin A}} \text{ ದತ್ತ ಸಮೀಕರಣದ ಎಡಭಾಗದ ಅಂಶ ಭೇದ ಎರಡನ್ನೂ } \sin A \text{ ಯಿಂದ ಭಾಗಿಸಿದೆ} \\ &= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \text{ ಅಂಶದಲ್ಲಿ } 1 \text{ ಎನ್ನುವಲ್ಲಿ } \operatorname{cosec}^2 A - \cot^2 A \text{ ಎಂದು ಆದೇಶಿಸಿದಾಗ.} \\ &= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A} = \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A + 1 - \operatorname{cosec} A} = \frac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{\cot A + 1 - \operatorname{cosec} A} \\ &= \cot A + \operatorname{cosec} A \end{aligned}$$

11.4.5.(vi)  $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$  ಎಂದು ಸಾಧಿಸಿ

$$\frac{(1 + \sin A)}{(1 - \sin A)} = \frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)} = \frac{(1 + \sin A)^2}{(1 - \sin^2 A)} = \frac{(1 + \sin A)^2}{\cos^2 A}$$

$$\therefore \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{(1 + \sin A)}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

11.4.5.(vii)  $\frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)} = \tan \theta$  ಎಂದು ಸಾಧಿಸಿ

$$\text{LHS} = \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)} = \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta [2(1 - \sin^2 \theta) - 1]} = \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (1 - 2 \sin^2 \theta)} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

11.4.5.(viii)  $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$  ಎಂದು ಸಾಧಿಸಿ

$$\text{LHS} = \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \cdot \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \cdot \sec A$$

$$= (\sin^2 A + \cos^2 A) + (1 + \tan^2 A) + 2 + (1 + \cot^2 A) + 2 = 7 + \tan^2 A + \cot^2 A$$

11.4.5.(ix)  $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$  ಎಂದು ಸಾಧಿಸಿ

$$\text{LHS} = \left(\frac{1}{\sin A} - \sin A\right) \left(\frac{1}{\cos A} - \cos A\right) = \left(\frac{1 - \sin^2 A}{\sin A}\right) \left(\frac{1 - \cos^2 A}{\cos A}\right) = \left(\frac{\cos^2 A}{\sin A}\right) \left(\frac{\sin^2 A}{\cos A}\right) = \cos A \cdot \sin A$$

$$\tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} = \frac{1}{\sin A \cos A} \quad \therefore \text{RHS} = \frac{1}{\tan A + \cot A} = \sin A \cdot \cos A \quad \therefore \text{LHS} = \text{RHS}$$

11.4.5.(x)  $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$  ಎಂದು ಸಾಧಿಸಿ

$$1 + \cot^2 A = 1 + \frac{1}{\tan^2 A} = \frac{\tan^2 A + 1}{\tan^2 A}$$

$$\text{LHS} = \frac{1 + \tan^2 A}{1 + \cot^2 A} = (1 + \tan^2 A) * \left\{ \frac{\tan^2 A}{1 + \tan^2 A} \right\} = \tan^2 A \quad \text{-----(1)}$$

$$(1 - \tan A)^2 = \left(1 - \frac{\sin A}{\cos A}\right)^2 = 1 + \left(\frac{\sin^2 A}{\cos^2 A}\right) - 2 \frac{\sin A}{\cos A} = \frac{\cos^2 A + \sin^2 A - 2 \sin A \cos A}{\cos^2 A} = \frac{1 - 2 \sin A \cos A}{\cos^2 A} \quad \text{-----(2)}$$

$$(1 - \cot A)^2 = \left(1 - \frac{\cos A}{\sin A}\right)^2 = 1 + \left(\frac{\cos^2 A}{\sin^2 A}\right) - 2 \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A - 2 \cos A \sin A}{\sin^2 A} = \frac{1 - 2 \cos A \sin A}{\sin^2 A} \quad \text{-----(3)}$$

$$(2) \div (3) \Rightarrow \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A \quad \text{-----(4)}$$

$$(1) = (4) \Rightarrow \frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$$

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