ಅಭ್ಯಾಸ 11.4

11.4.1. sinA, secA ಮತ್ತು tanA ಈ ತ್ರಿಕೋನಮಿತಿ ಅನುಪಾತಗಳನ್ನು cotA ರೂಪದಲ್ಲಿ ವ್ಯಕ್ತಪಡಿಸಿ.

sinA	secA	cotA
$1= \sin^2A+\cos^2A$ ಇದರ ಎರಡೂ ಬದಿಯನ್ನು $\sin^2A$ ಯಿಂದ ಭಾಗಿಸಿ $\frac{1}{\sin^2 A}=1+\frac{\cos^2 A}{\sin^2 A}=1+\cot^2A$ $\sin^2A=\frac{1}{1+\cot^2 A}$ $\therefore \sin A = \pm\frac{1}{\sqrt{1+\cot^2 A}}$	$1= \sin^2A+\cos^2A$ ಇದರ ಎರಡೂ ಬದಿಯನ್ನು $\cos^2A$ ಯಿಂದ ಭಾಗಿಸಿ $\frac{1}{\cos^2 A}=\frac{\sin^2 A}{\cos^2 A}+1$ $\frac{1}{\cos^2 A}=\tan^2A+1=\frac{1}{\cot^2 A}+1$ $\text{Sec}^2A=\frac{\cot^2 A+1}{\cot^2 A}$ $\therefore \sec A = \pm\frac{\sqrt{\cot^2 A+1}}{\cot A}$	$\tan A=\frac{1}{\cot A}$

11.4.2. $\angle A$ ದ ಎಲ್ಲಾ ತ್ರಿಕೋನಮಿತಿ ಅನುಪಾತಗಳನ್ನು sec A ರೂಪದಲ್ಲಿ ಬರೆಯಿರಿ

sinA & cosecA =	cosA=	tanA & cotA=
$\sin^2A=1-\cos^2A$ $=1-\frac{1}{\sec^2 A}=\frac{\sec^2 A-1}{\sec^2 A}$ $\therefore \sin A=\pm\frac{\sqrt{\sec^2 A-1}}{\sec A}$ $\Rightarrow \text{cosec}A=\frac{1}{\sin A}=\pm\frac{\sec A}{\sqrt{\sec^2 A-1}}$	$\frac{1}{\sec A}$	$1= \sin^2A+\cos^2A$ $\frac{1}{\cos^2 A}=\frac{\sin^2 A}{\cos^2 A}+1$ $\text{Sec}^2A = \tan^2A+1$ $\Rightarrow \tan^2A =\sec^2A-1$ $\tan A=\sqrt{\sec^2 A-1}\Rightarrow \cot A=\frac{1}{\tan A}=\frac{1}{\sqrt{\sec^2 A-1}}$

ಪ್ರಶ್ನೆ	ಬೆಲೆಯನ್ನು ಕಂಡುಹಿಡಿಯಿರಿ/ಉತ್ತರಿಸಿ
11.4.5.(i)	$\frac{\sin^2 63^O + \sin^2 27^O}{\cos^2 17^O + \cos^2 73^O}$ = **??** $\sin^2 63^O+\sin^2 27^O=\sin^2(90^O-27^O)+\sin^2 27^O=\cos^2 27^O+\sin^2 27^O=1$ -----(1) $\cos^2 17^O+\cos^2 73^O=\cos^2(90^O-73^O)+\cos^2 73^O=\sin^2 73^O+\cos^2 73^O=1$ -----(2) (1) ÷ (2) $\Rightarrow \frac{\sin^2 63^O + \sin^2 27^O}{\cos^2 17^O + \cos^2 73^O}=1$
11.4.5.(ii)	sin 25°cos 65°+cos 25°sin 65° = **??** $=\sin 25^O*\cos 65^O+\cos 25^O*\sin 65^O$ $=\sin 25^O*\cos(90^O-25^O)+\cos 25^O*\sin(90^O-25^O)$ $=\sin 25^O*\sin 25^O+\cos 25^O*\cos 25^O=\sin^2 25^O+\sin^2 25^O=1$
11.4.4.(i)	$9\sec^2 A - 9\tan^2 A=$ **??** $= 9(\frac{1}{\cos^2 A}-\frac{\sin^2 A}{\cos^2 A})=9(\frac{1-\sin^2 A}{\cos^2 A})=9(\frac{\cos^2 A}{\cos^2 A})=9$
11.4.4.(ii)	$(1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\theta)=$ **??** $=(1+\tan\theta+\sec\theta)+(\cot\theta+\cot\theta*\tan\theta+\cot\theta.\sec\theta)-(\mathrm{cosec}\theta+\mathrm{cosec}\theta.\tan\theta+\mathrm{cosec}\theta.\sec\theta)$ $=(1+\tan\theta+\sec\theta)+(\cot\theta+1+\mathrm{cosec}\theta)-\mathrm{cosec}\theta-\sec\theta-\mathrm{cosec}\theta.\sec\theta$ $[\because \cot\theta.\sec\theta=\frac{\cos\theta}{\sin\theta}*\frac{1}{\cos\theta}=\mathrm{cosec}\theta$ & $\mathrm{cosec}\theta.\tan\theta=\frac{1}{\sin\theta}*\frac{\sin\theta}{\cos\theta}=\sec\theta]$ $=2+\tan\theta+\cot\theta-\mathrm{cosec}\theta.\sec\theta$ $=2+\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}-\mathrm{cosec}\theta.\sec\theta=2+\frac{\sin^2\theta+\cos^2\theta}{\cos\theta*\sin\theta}-\mathrm{cosec}\theta.\sec\theta$ $=2+\frac{1}{\cos\theta*\sin\theta}-\mathrm{cosec}\theta.\sec\theta=2+\sec\theta.\mathrm{cosec}\theta-\mathrm{cosec}\theta.\sec\theta=2$
11.4.4.(iii)	$\frac{1+\tan^2 A}{1+\cot^2 A}$ = **??** $1+\cot^2 A= 1+\frac{1}{\tan^2 A}=\frac{\tan^2 A+1}{\tan^2 A}$ $\frac{1+\tan^2 A}{1+\cot^2 A}=(1+\tan^2 A)*\{\frac{\tan^2 A}{1+\tan^2 A}\}=\tan^2 A$

11.4.5.(i) $(\text{cosec } \theta - \cot \theta)^2 = \frac{(1-\cos\theta)}{(1+\cos\theta)}$ ಎಂದು ಸಾಧಿಸಿ

$$(\text{cosec}\,\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} = \frac{(1-\cos\theta)}{(1+\cos\theta)}$$

11.4.5.(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$ ಎಂದು ಸಾಧಿಸಿ

$$\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)*\cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)*\cos A} = \frac{1+1+2\sin A}{(1+\sin A)*\cos A} = \frac{2(1+\sin A)}{(1+\sin A)*\cos A} = \frac{2}{\cos A} = 2\sec A$$

11.4.5.(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \text{cosec}\,\theta.\sec\theta$ ಎಂದು ಸಾಧಿಸಿ

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta * \tan\theta}{\tan\theta - 1} + \frac{\cot\theta}{1-\tan\theta} \left(\because 1-\cot\theta = 1 - \frac{1}{\tan\theta} = \frac{\tan\theta - 1}{\tan\theta} \Rightarrow \frac{1}{1-\cot\theta} = \frac{\tan\theta}{\tan\theta - 1}\right)$$

$$= \frac{\tan^2\theta - \cot\theta}{\tan\theta - 1} = \frac{\tan^2\theta - \frac{1}{\tan\theta}}{\tan\theta - 1} = \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)} = \frac{(\tan\theta - 1)(\tan^2\theta + 1 + \tan\theta)}{\tan\theta(\tan\theta - 1)} = \frac{(\tan^2\theta + 1 + \tan\theta)}{\tan\theta} = \tan\theta + \frac{1}{\tan\theta} + 1$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} + 1 = 1 + \frac{\sin^2\theta + \cos^2\theta}{\cos\theta * \sin\theta} = 1 + \frac{1}{\cos\theta * \sin\theta} = 1 + \text{cosec}\,\theta.\sec\theta$$

11.4.5.(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{(1-\cos A)}$ ಎಂದು ಸಾಧಿಸಿ

$$\text{LHS} = 1 + \frac{1}{\sec A} = 1 + \cos A = \frac{(1+\cos A)(1-\cos A)}{(1-\cos A)} = \frac{(1-\cos^2 A)}{(1-\cos A)} = \frac{\sin^2 A}{(1-\cos A)}$$

11.4.5.(v) 1=cosec^{2}A-cot^{2}A ಎನ್ನುವ ನಿತ್ಯ ಸಮೀಕರಣ ಉಪಯೋಗಿಸಿ $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$ =cosecA+cotA ಎಂದು ಸಾಧಿಸಿ

LHS= $\frac{\frac{\cos A-\sin A+1}{\sin A}}{\frac{\cos A+\sin A-1}{\sin A}}$ ದತ್ತ ಸಮೀಕರಣದ ಎಡಭಾಗದ ಅಂಶ ಛೇದ ಎರಡನ್ನೂ sinA ಯಿಂದ ಭಾಗಿಸಿದೆ

$=\frac{\cot A-1+\operatorname{cosec} A}{\cot A+1-\operatorname{cosec} A}$ ಅಂಶದಲ್ಲಿ 1 ಎನ್ನುವಲ್ಲಿ cosec^{2}A-cot^{2}A ಎಂದು ಆದೇಶಿಸಿದಾಗ.

$$=\frac{\cot A+\operatorname{cosec} A-(\operatorname{cosec}^2 A-\cot^2 A)}{\cot A+1-\operatorname{cosec} A}=\frac{\cot A+\operatorname{cosec} A-(\operatorname{cosec} A+\cot A)(\operatorname{cosec} A-\cot A)}{\cot A+1-\operatorname{cosec} A}=\frac{(\cot A+\operatorname{cosec} A)(1-\operatorname{cosec} A+\cot A)}{\cot A+1-\operatorname{cosec} A}$$

=cotA+cosecA

11.4.5.(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}$ =secA+tanA ಎಂದು ಸಾಧಿಸಿ

$$\frac{(1+\sin A)}{(1-\sin A)}=\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}=\frac{(1+\sin A)^2}{(1-\sin^2 A)}=\frac{(1+\sin A)^2}{\cos^2 A}$$

$\therefore \sqrt{\frac{1+\sin A}{1-\sin A}}=\frac{(1+\sin A)}{\cos A}=\frac{1}{\cos A}+\frac{\sin A}{\cos A}$ =secA+tanA

11.4.5.(vii) $\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$ = tanθ ಎಂದು ಸಾಧಿಸಿ

LHS= $\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}=\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta[2(1-\sin^2\theta)-1]}=\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(1-2\sin^2\theta)}=\frac{\sin\theta}{\cos\theta}$ = tanθ

11.4.5.(viii) $(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2=7+\tan^2A+\cot^2A$ ಎಂದು ಸಾಧಿಸಿ

LHS $=\sin^2A+\operatorname{cosec}^2A+2\sin A.\operatorname{cosec}A+\cos^2A+\sec^2A+2\cos A.\sec A$

$=(\sin^2A+\cos^2A)+(1+\tan^2A)+2+(1+\cot^2A)+2=7+\tan^2A+\cot^2A$

11.4.5.(ix) $(\operatorname{cosec}A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$ ಎಂದು ಸಾಧಿಸಿ

$\text{LHS}=\left(\dfrac{1}{\sin A}-\sin A\right)\left(\dfrac{1}{\cos A}-\cos A\right)=\left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right)=\left(\dfrac{\cos^2 A}{\sin A}\right)\left(\dfrac{\sin^2 A}{\cos A}\right)=\cos A.\sin A$

$\tan A+\cot A=\dfrac{\sin A}{\cos A}+\dfrac{\cos A}{\sin A}=\dfrac{\sin^2 A+\cos^2 A}{\sin A\cos A}=\dfrac{1}{\sin A\cos A}$ $\therefore$ RHS $=\dfrac{1}{\tan A+\cot A}=\sin A.\cos A$ $\therefore$ LHS=RHS

11.4.5.(x) $\dfrac{1+\tan^2 A}{1+\cot^2 A}=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2A$ ಎಂದು ಸಾಧಿಸಿ

$1+\cot^2A=1+\dfrac{1}{\tan^2 A}=\dfrac{\tan^2 A+1}{\tan^2 A}$

$\text{LHS}=\dfrac{1+\tan^2 A}{1+\cot^2 A}=(1+\tan^2A)*\left\{\dfrac{\tan^2 A}{1+\tan^2 A}\right\}=\tan^2A$ -----------------(1)

$(1-\tan A)^2=\left(1-\dfrac{\sin A}{\cos A}\right)^2=1+\left(\dfrac{\sin^2 A}{\cos^2 A}\right)-2\dfrac{\sin A}{\cos A}=\dfrac{\cos^2 A+\sin^2 A-2\sin A\cos A}{\cos^2 A}=\dfrac{1-2\sin A\cos A}{\cos^2 A}$ -------------(2)

$(1-\cot A)^2=\left(1-\dfrac{\cos A}{\sin A}\right)^2=1+\left(\dfrac{\cos^2 A}{\sin^2 A}\right)-2\dfrac{\cos A}{\sin A}=\dfrac{\sin^2 A+\cos^2 A-2\cos A\sin A}{\sin^2 A}=\dfrac{1-2\cos A\sin A}{\sin^2 A}$ -------------(3)

$(2)\div(3)\Rightarrow\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\dfrac{\sin^2 A}{\cos^2 A}=\tan^2A$ ----------(4)

$(1)=(4)\Rightarrow\dfrac{1+\tan^2 A}{1+\cot^2 A}=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2A$